KS Nachrichtentechnik

Monday 22 February, 2021

700.171 - WS 2020

Written Exam

Answer the following questions. Total points: 125. Sufficient if ≥ 60. Time 2 hours.

1. **Convolution, Fourier Series, Fourier Transform** (55 points)
   - Let $y(t)$ be defined as $y(t) = \underbrace{x * x * \cdots * x(t)}_{n \text{ times}}$ with $x(t) = \text{rect}(t)$ and $n = 3$:
     (i) Compute the Fourier Transform $Y(f)$ and sketch it. (12)
     (ii) Evaluate the area of $y(t)$. (6)
     (iii) Evaluate the area of $Y(f)$. (10)
     (iv) Let $z(t) = y * h(t)$ with $h(t) = B\text{sinc}(Bt)$. What is the minimum sampling frequency F_s to guarantee a perfect reconstruction from the sampled version $z(kT)$? (7)
     (v) Find the closed-form expression for $y(t)$ and sketch it. (12)
     (vi) Evaluate the energy of $y(t)$. (8)

2. **Complex Representation, Analytic Signal, Hilbert Transform** (35 points)
   - Let $x(t)$ be a continuous-time real signal with spectrum given as in Fig. 1:
     (i) Compute its analytic signal $x^{(a)}(t)$. (22)
     (ii) Compute its Hilbert Transform $x_H(t)$. (5)
     (iii) Evaluate the energy of $x_H(t)$. (8)

3. **Random Variables, Random Process Theory, Power Spectral Density** (35 points)
   - Let $X(t)$ be a Gaussian process with zero mean and autocorrelation function $r_{XX}(\tau) = \frac{R_0}{T}\text{sinc}^2(\frac{\tau}{T})$. The process is filtered through an interpolation filter with impulse response $h(t) = \frac{1}{T}\text{sinc}(\frac{t}{T})$ so that $Y(t) = X * h(t)$:
     (i) Evaluate the average power M_X. (5)
     (ii) Calculate the power spectral density of $X(t)$. (8)
     (iii) Calculate the power spectral density of $Y(t)$ and sketch it. (8)
     (iv) Evaluate the average power M_Y. (10)
     (v) Evaluate the probability $P_1 = P[|Y(t)| < \sqrt{\frac{3R_0}{4T}}]$. (4)

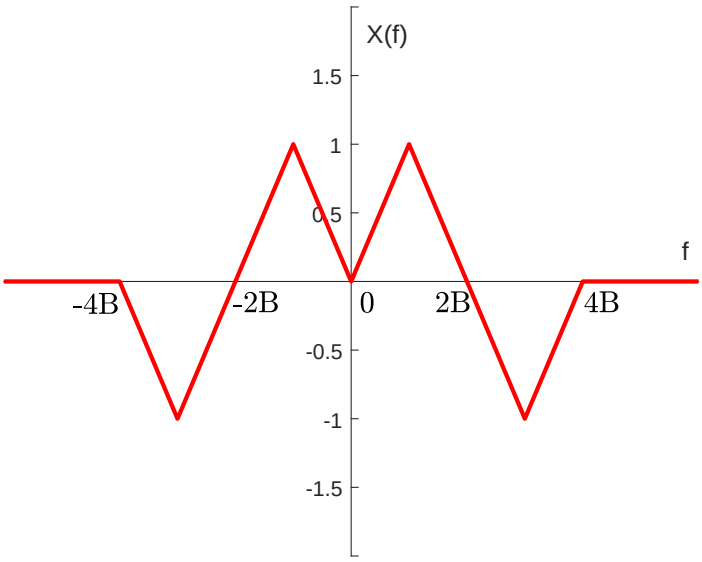

Figure 1: Spectrum $X(f)$ of exercise 2.