KS Nachrichtentechnik

Tuesday 26 January, 2021

700.171 - WS 2020

Written Exam Simulation

Answer the following questions. Total points: 100. Sufficient if ≥ 60. Time 1.5 hours.

1. **Convolution, Fourier Series, Fourier Transform** (40 points)
   - Let $x_p(t)$ be the periodic version of $x(t) = \text{sgn}(t)$ with period $T_0 = 2$:
     (i) Compute the Fourier Series of $x_p(t)$. (15)
     (ii) Having found that
     $$x_p(t) = \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1-(-1)^k}{k} \sin(\pi k t),$$
     evaluate the series $\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$. (5)
     (iii) Compute the Fourier Transform $X_w(f)$ of the windowed signal $x_w(t) = x(t) \cdot \text{rect}(t/2)$ and sketch the magnitude. (10)
     (iv) What is the minimum sampling frequency F_s to guarantee a perfect reconstruction from the samples $x_w(kT)$? (5)
     (v) Let $X_s(h) = X_w(hF)$ be the sampled version of $X_w(f)$, with $h \in \mathbb{Z}$ and $F = 1/2$. How is $X_s(h)$ related with $x_p(t)$? (5)

2. **Complex Representation, Analytic Signal, Hilbert Transform** (30 points)
   - Let $x(t) = \text{sinc}^2(t) \cdot \cos(2\pi t)$ be a continuous-time signal:
     (i) Compute its analytic signal $x^{(a)}(t)$. (15)
     (ii) Compute its Hilbert Transform $x_H(t)$ and sketch it. (10)
     (iii) Evaluate the energy of $x_H(t)$. (5)

3. **Random Variables, Random Process Theory, Power Spectral Density** (30 points)
   - Let $X(t)$ be a continuous-time WSS random process with zero mean and autocorrelation function $r_{XX}(\tau) = e^{-|\tau|}$.
     (i) Evaluate the average power M_X. (5)
     (ii) Calculate the power spectral density of $X(t)$. (15)
     (iii) Determine the band B such as the power inside the spectral extension $[-B, B]$ is half of the total power M_X . (10)