KS Nachrichtentechnik

Monday 14 February, 2022

700.171 - WS 2021

Written Exam

Answer the following questions. Total points: 120. Sufficient if ≥ 60. Time 2 hours.

1. **Convolution, Fourier Series, Fourier Transform** (50 points)
   - Let $x(t)$ be defined as $x(t) = \cos(2\pi t) \cdot \text{rect}(t)$.
     (i) Evaluate the area and energy of $x(t)$. (10)
     (ii) Compute the Fourier Series of the periodic version $x_p(t)$

$$x_p(t) = \sum_{h=-\infty}^{+\infty} x(t - hT_0),$$

     with period $T_0 = 2$. (20)
     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$. (15)
     (iv) Sketch $X(f)$. (5)

2. **Complex Representation, Analytic Signal, Hilbert Transform** (35 points)
   - Let $x(t)$ be a continuous-time real signal defined as

$$x(t) = \frac{\sin(2\pi t) \cdot \cos(4\pi t)}{6\pi t}.$$

     (i) Compute its analytic signal $x^{(a)}(t)$. (15)
     (ii) Compute the Hilbert Transform $x_H(t)$ of $x(t)$. (5)
     (iii) Evaluate the energy of $x_H(t)$. (8)
     (iv) Compute the Hilbert Transform $x_{HH}(t)$ of $x_H(t)$. (7)

3. **Random Process Theory, Power Spectral Density** (35 points)
   - Let $X(t)$ be a WSS Gaussian process with mean $m_X = \sqrt{\frac{R_0}{4T}}$ and autocorrelation function $r_{XX}(\tau) = \frac{R_0}{2T}\text{sinc}(\frac{\tau}{2T})$. The process is filtered through an interpolation filter with impulse response $h(t) = \frac{1}{T}\text{sinc}(\frac{t}{T})$ so that $Y(t) = X * h(t)$:
     (i) Evaluate the average power M_X. (5)
     (ii) Evaluate the mean m_Y of $Y(t)$. (5)
     (iii) Compute the power spectral density of $Y(t)$ and the power M_Y. (15)
     (iv) Evaluate the probability $P_1 = P[Y(t) < \sqrt{\frac{R_0}{4T}}]$. (10)