**KS Nachrichtentechnik**

Friday 20 January, 2023

**700.171 - WS 2022**

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**Written Exam**

Answer the following questions. Total points: 120. Sufficient if $\geq 60$. Time 2 hours.

1. **Convolution, Fourier Series, Fourier Transform** (55 points)
   - Let $x(t)$ be defined as $x(t) = |\cos(\pi t) + \frac{1}{2}|$.
     (i) Sketch $x(t)$ and find the period $T_0$. (5)
     (ii) Evaluate the area and energy of $x(t)$ in one period. (10)
     (iii) Compute the Fourier Transform $Y(f)$ of $y(t) = x(t) \cdot \text{rect}\left(\frac{3t}{4}\right)$. (20)
     (iv) Compute the Fourier Series of $x(t)$. (20)

2. **Complex Representation, Analytic Signal, Hilbert Transform** (30 points)
   - Let $x(t)$ be a continuous-time real signal defined as a convolution

$$x(t) = \text{sinc}(t) * \text{sinc}(3t).$$

     (i) Compute its analytic signal $x^{(a)}(t)$. (15)
     (ii) Compute the Hilbert Transform $x_H(t)$ of $x(t)$. (7)
     (iii) Evaluate the energy of $x_H(t)$. (8)

3. **Random Variables, Random Process Theory, Power Spectral Density** (35 points)
   - Let $X(t)$ be a continuous-time WSS random process with zero mean and autocorrelation function $r_{XX}(\tau) = e^{-|\tau|}$.
     (i) Evaluate the average power $M_X$. (5)
     (ii) Calculate the power spectral density of $X(t)$. (15)
     (iii) Determine the band $B$ such as the power inside the spectral extension $[-B, B]$ is half of the total power $M_X$. (15)

     It is useful to know that

$$\int \frac{1}{1+x^2} \mathrm{d}x = \arctan(x) + C$$