KS Nachrichtentechnik

Wednesday 21 February, 2024

700.172 - WS 2023

Written Exam

Answer the following questions. Total points: 120. Sufficient if ≥ 60. Time 2 hours.

1. **Convolution, Fourier Series, Fourier Transform** (40 points)
   - Let $x(t)$ be defined as $x(t) = \text{triangle}\big(\text{rect}(t)\big)$.
     (i) Evaluate the area and energy in one period of the periodic version $x_p(t)$

$$x_p(t) = \sum_{h=-\infty}^{+\infty} x(t - hT_0) \cdot \text{rect}\left(\frac{t - hT_0}{T_0}\right),$$

     with period $T_0 = 2$. (5)

     (ii) Compute the Fourier Series of $x_p(t)$. (15)

     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$. (10)

     (iv) Compute the Inverse Fourier Transform $y(t)$ of $Y(f) = X(f) \cdot \pi f$. (10)

2. **Complex Representation, Analytic Signal, Hilbert Transform** (50 points)
   - Let $x(t)$ be a continuous-time real signal with spectrum ($X(f)$) given as in Fig. 1:
     (i) Compute its analytic signal $x^{(a)}(t)$. (20)

     (ii) Compute the Hilbert Transform $x_H(t)$ of $x(t)$. (10)

     (iii) Evaluate the energy of $x_H(t)$. (20)

3. **Random Process Theory, Power Spectral Density** (30 points)
   - Let $X(t)$ be a WSS Gaussian process with zero mean and autocorrelation function $r_{XX}(\tau) = \text{sinc}(\tau)$. The processed is filtered through an interpolation filter with impulse response $h(t) = \frac{1}{T}\text{sinc}\big(\frac{t}{T}\big)$ so that $Y(t) = X * h(t)$:
     (i) Evaluate the average power M_X. (5)

     (ii) Compute the power spectral density of $Y(t)$. (15)

     (iii) Determine the value T such as the power at the output is half of the total power M_X. (10)

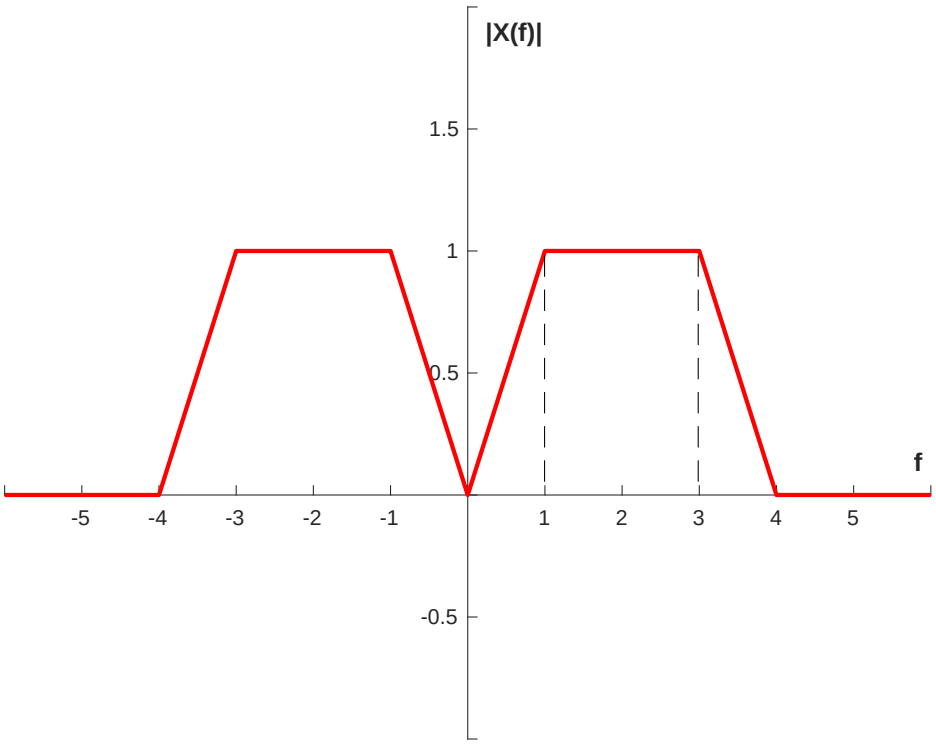

Figure 1: Spectrum $X(f)$ of exercise 2.