KS Nachrichtentechnik

Wednesday 31 January, 2024

700.172 - WS 2023

Simulation Exam

Answer the following questions. Total points: 120. Sufficient if ≥ 60. Time 2 hours.

1. **Convolution, Fourier Series, Fourier Transform** (50 points)
   - Let $x(t)$ be defined as $x(t) = \text{triangle}(t) \cdot \text{rect}(t - 1/2)$.
     (i) Evaluate the area and energy of $x(t)$. (5)
     (ii) Compute the Fourier Series of the periodic version $x_p(t)$

$$x_p(t) = \sum_{h=-\infty}^{+\infty} x(t - hT_0),$$

     with period $T_0 = 2$. (15)
     (iii) Compute the Fourier Transform $X_+(f)$ of $x(t) + x(-t)$. (10)
     (iv) Compute the Fourier Transform $X_-(f)$ of $x(t) - x(-t)$. (10)
     (v) Compute the Fourier Transform $X(f)$ of $x(t)$. (10)

2. **Complex Representation, Analytic Signal, Hilbert Transform** (35 points)
   - Let $x(t)$ be a continuous-time real signal defined as

$$x(t) = \text{sinc}^3(t) \cdot \cos(6\pi f_0 t).$$

     (i) Compute its analytic signal $x^{(a)}(t)$. (15)
     (ii) Compute the Hilbert Transform $x_H(t)$ of $x(t)$. (5)
     (iii) Evaluate the energy of $x_H(t)$. (15)

3. **Random Process Theory, Power Spectral Density** (35 points)
   - Let $X(t)$ be a continuous-time WSS random process with zero mean and autocorrelation function $r_{XX}(\tau) = e^{-|\tau|}$.
     (i) Evaluate the average power M_X. (5)
     (ii) Calculate the power spectral density of $X(t)$. (15)
     (iii) Determine the band B such as the power inside the spectral extension $[-B, B]$ is half of the total power M_X. (15)

     It is useful to know that

$$\int \frac{1}{1+x^2} \mathrm{d}x = \arctan(x) + C$$