Digitale Signalverarbeitung
Wednesday 17 June, 2020

700.121 - SS 2020

Written Exam

Answer the following questions. Total points: 110. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Series, Fourier Transform, Sampling Theorem** (35 points)
   - Let $x_p(t)$ be the periodic version of $x(t) = \text{triangle}(t)$ with period $T_0 = 2$:
     (i) Compute the Fourier Series of $x_p(t)$. (15)
     (ii) Having found that
     $$x_p(t) = \frac{1}{2} + \frac{2}{\pi^2} \sum_{k=1}^{\infty} \frac{1-(-1)^k}{k^2} \cos(\pi k t),$$
     evaluate the series $\sum_{k=1}^{\infty} \frac{1}{(2k-1)^4}$. (7)
     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$ and sketch the magnitude. (5)
     (iv) What is the minimum sampling frequency F_c to guarantee a perfect reconstruction from the samples $x(kT)$? (3)
     (v) How is related the coefficient A_k of the Fourier Series with the Fourier Transform $X(f)$? (5)

2. **Discrete Fourier Transform** (25 points)
   - An N-point time signal $x(n)$ is defined as
   $$x(n) = \underbrace{[1, -1, 1, -1, \ldots, (-1)^{N-2}, (-1)^{N-1}]}_{N \text{ values}}, \quad 0 \leq n \leq N-1$$
     (i) Find the 2-point DFT of $x(n) = [1, -1]$ and express the result in polar coordinates. (5)
     (ii) Find the N-point DFT $X(h)$ and express the result in algebraic coordinates. (15)
     (iii) Let $x_f(n)$ be the flipped version of $x(n)$. Find the DFT $X_f(h)$. (5)

3. **Z-Transform, Discrete Time System, Power Spectral Density** (50 points)
   - A simple discrete time system is described by the following input-output relation:
   $$y(n) = \alpha \cdot y(n-1) + x(n) - x(n-1)$$
   where $x(n)$ is the signal to filter. Assume everywhere $\alpha = 0.5$.

(i) Find the transfer function $H(z)$ and sketch the zero/pole diagram. (12)

(ii) Find the impulse response $h(n)$ of the filter and sketch it. (6)

(iii) Find the frequency response $\hat{H}(f)$ and sketch its magnitude. What type of filter is it? (10)

(iv) Discuss the behavior of the filter when $|\alpha| \to 1$. (7)

(v) The random input signal $x(n)$ consists of independent, zero mean, unit variance Gaussian distributed samples. Find the power spectral density $R_{yy}(f)$ of the output $y(n)$. (5)

(vi) Find the power M_y at the system's output. (10)