Digitale Signalverarbeitung
Friday 12 June, 2020

700.121 - SS 2020

Exam simulation

Answer the following questions. Total points: 100. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Transform, Sampling Theorem** (35 points)
   - Let $x(t) = \text{sinc}^n(2-3t)$ be a continuous-time signal with $n = 2$:
     (i) Sketch the amplitude of $x(t)$. (5)
     (ii) Compute its Fourier Transform $X(f)$ and sketch the magnitude. (12)
     (iii) Evaluate the energy of $X(f)$. (10)
     (iv) What is the minimum sampling frequency F_c to guarantee a perfect reconstruction from the samples $x(kT)$? (3)
     (v) If $n = 3$, what would be the new minimum sampling frequency F_c? (5)

2. **Discrete Fourier Transform** (15 points)
   - The DFT on an N-point signal $x(n)$ is given by

$$X(h) = [0, e^{j\theta}, \underbrace{0, 0, \ldots, 0}_{N-3 \text{ values}}, e^{-j\theta}], \quad 0 \leq h \leq N-1$$

     (i) Find the N-point signal $x(n)$ and simplify your answer. (10)
     (ii) Let $X_f(h)$ be the flipped version of $X(h)$. Find the IDFT $x_f(n)$. (5)

3. **Z-Transform, Discrete Time System, Power Spectral Density** (50 points)
   - A very simple high-pass digital filtering can be performed using the following FIR filter:

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} (-1)^k x(n-k)$$

     where $x(n)$ is the signal to filter. Assume $N = 8$.
     (i) Sketch the impulse response $h(n)$ of the high-pass filter. (5)
     (ii) What is the DC gain ($\hat{H}(0)$) of the filter? (5)
     (iii) Find the transform function $H(z)$ and sketch the zero/pole diagram. (12)
     (iv) Find the frequency response $\hat{H}(f)$ and sketch its magnitude. (10)
     (v) The random input signal $x(n)$ is a moving average between two consecutive independent, zero mean, unit variance Gaussian distributed samples $w(n)$. Find the power spectral density $R_{yy}(f)$ of the output $y(n)$. (10)
     (vi) Find the power M_y at the system's output. (8)