**Digitale Signalverarbeitung**
Thursday 15 July, 2021

**700.121 - SS 2021**

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**Written Exam**
Answer the following questions. Total points: 110. Sufficient if $\geq 60$. Time 1.5 hours.

1. **Fourier Series, Fourier Transform, Sampling Theorem** (40 points)
   - Let $x_p(t)$ be the periodic version of the signal illustrated in Fig.1, with $D \leq 1$ and period $T_0 = 2$:
     (i) Compute the Fourier Series of $x_p(t)$. (15)
     (ii) Having found that

$$x_p(t) = \frac{D}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\sin\left(\frac{\pi Dk}{2}\right)}{k} \cos\left(\pi kt - \frac{\pi Dk}{2}\right),$$

     prove that

$$\sum_{k=1}^{\infty} \frac{\sin^2\left(\frac{\pi Dk}{2}\right)}{k^2} = \left(D - \frac{D^2}{2}\right)\frac{\pi^2}{4}. \quad (7)$$

     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$ and sketch magnitude and phase. (5)
     (iv) Find $y(t) = x * x(t)$ the convolution of $x$ with itself and evaluate the Fourier Transform $Y(f)$. (13)

2. **Discrete Fourier Transform** (30 points)
   - A 4-point triangular shaped time signal $x(n)$ is defined as

$$x(n) = [0, 1, 2, 1].$$

     (i) Find the 2-point DFT of the 2-point signal $y(n)$ defined as $y(n) = [x(1), x(3)]$ and express the result in polar coordinates. (7)
     (ii) Find the 4-point DFT $X(h)$ of $x(n)$ and express the result in algebraic coordinates. (15)
     (iii) Find $\alpha$ and $\beta$ that verify

$$X(h) = 4\cos(\alpha h)\cos(\beta h). \quad (8)$$

3. **Z-Transform, Discrete Time System, Power Spectral Density** (40 points)

- A simple discrete time system is described by the following input-output relation:

$$y(n) = y(n-2) + x(n) - x(n-2)$$

where $x(n)$ is the signal to filter.

(i) Find the transfer function $H(z)$ and sketch the zero/pole diagram. (10)

(ii) Find the impulse response $h(n)$ of the filter and sketch it. (8)

(iii) Find the frequency response $\hat{H}(f)$ and sketch its magnitude. What type of filter is it? (8)

(iv) The random input signal $x(n)$ consists of independent, zero mean, Gaussian distributed samples with power spectral density $R_{xx}(f) = \text{rect}(4f - 1)$. Find the power spectral density $R_{yy}(f)$ of the output $y(n)$. (6)

(vi) Find the power $M_x$ and $M_y$ at the system's input and output, respectively. (8)

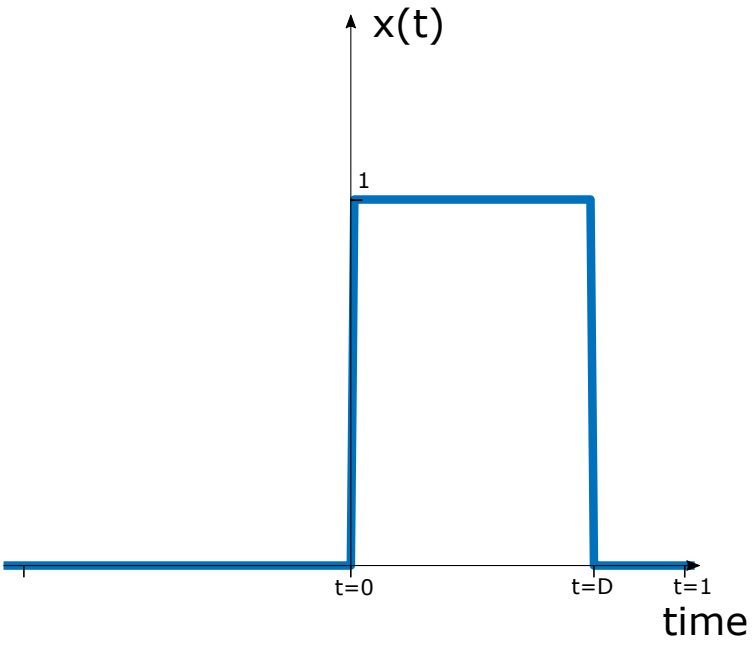


Figure 1: Signal $x(t)$