Digitale Signalverarbeitung
Tuesday 29 June, 2021

700.121 - SS 2021

Exam simulation
Answer the following questions. Total points: 100. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Series, Fourier Transform, Sampling Theorem** (30 points)
   - Let $x_p(t)$ be the periodic version of the signal $x(t)$ illustrated in Fig.1, with $D \leq 2$ and period $T_0 = 2$:
     (i) Compute the Fourier Series of $x_p(t)$. (10)
     (ii) Having found that
     $$x_p(t) = \frac{D}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\sin\left(\frac{\pi D k}{2}\right)}{k} \cos(\pi k t),$$
     evaluate the series $\sum_{k=1}^{\infty} \frac{\sin\left(\frac{\pi D k}{2}\right)}{k}$. (7)
     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$ and sketch the magnitude. (5)
     (iv) What is the minimum sampling frequency F_s to guarantee a perfect reconstruction from the samples $x(kT)$? (3)
     (v) How is related the coefficient A_k of the Fourier Series with the Fourier Transform $X(f)$? (5)

2. **Discrete Fourier Transform** (25 points)
   - An N-point time signal $x(n)$ is defined as
     $$x(n) = 2^{-n},$$
     where $0 \leq n \leq N-1$.
     (i) Find the 2-point DFT of $x(n) = [1, 0.5]$ and express the result in polar coordinates. (7)
     (ii) Find the N-point DFT $X(h)$ and express the result in algebraic coordinates. (18)

3. **Z-Transform, Discrete Time System, Power Spectral Density** (45 points)
   - A simple discrete time system is described by the following input-output relation:
     $$y(n) = \frac{1}{4} y(n-2) + x(n)$$
     where $x(n)$ is the signal to filter.

(i) Find the transfer function $H(z)$ and sketch the zero/pole diagram. (12)

(ii) Find the impulse response $h(n)$ of the filter and sketch it. (8)

(iii) Find the frequency response $\hat{H}(f)$ and sketch its magnitude. What type of filter is it? (10)

(iv) The random input signal $x(n)$ is itself the output of a filtering process $x(n) = h_2 * w(n)$ with $h_2(n) = \delta(n) - 0.25 \cdot \delta(n-2)$ and where $w(n)$ consists of independent, zero mean, unit-variance Gaussian distributed samples. Find the power spectral density $R_{yy}(f)$ of the output $y(n)$. (15)

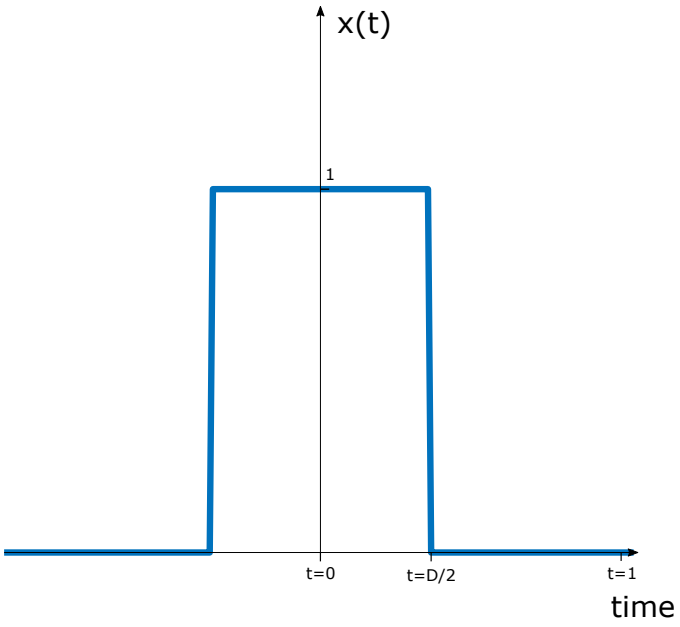

Figure 1: Signal $x(t)$.