Digitale Signalverarbeitung
Friday 1 July, 2022

700.121 - SS 2022

Written Exam

Answer the following questions. Total points: 110. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Series, Fourier Transform, Sampling Theorem** (40 points)
   - Let $x_p(t)$ be the periodic version of the signal illustrated in Fig.1, with $D \leq 1$ and period $T_0 = 2$:
     (i) Compute the Fourier Series of $x_p(t)$. (15)
     (ii) Having found that

$$x_p(t) = \frac{1+D}{2} + \frac{2}{(1-D)\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1} + \cos(\pi k D)}{k^2} \cos(\pi k t),$$

     prove that

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} = \frac{\pi^2}{8} \quad (5)$$

     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$. (15)
     (iv) How is related the coefficient A_k of the Fourier Series with the Fourier Transform $X(f)$? (5)

2. **Discrete Fourier Transform** (30 points)
   - An N-point time signal $x_N(n)$ is defined as

$$x_N(n) = [1, 1, \underbrace{0, 0, \ldots, 0}_{N-4 \text{ zero values}}, 1, 1], \quad 0 \leq n \leq N-1, N \geq 4$$

     (i) Find the 4-point DFT of the 4-point signal $y(n)$ defined as $y(n) = x_4(n)$ and express the result in polar coordinates. (7)
     (ii) Find the N-point DFT $X(h)$ of $x_N(n)$. (15)
     (iii) Express the previous result in polar coordinates and find α and β that verify

$$|X(h)| = 4\left|\cos\left(\alpha\frac{\pi h}{N}\right)\cos\left(\beta\frac{\pi h}{N}\right)\right|. \quad (8)$$

3. **Z-Transform, Discrete Time System, Power Spectral Density** (40 points)

- A simple discrete-time system is described by the following input-output relation:

$$y(n) = y(n-1) + x(n) - x(n-2)$$

where $x(n)$ is the signal to filter.

(i) Find the transfer function $H(z)$ and sketch the zero/pole diagram. (10)

(ii) Find the impulse response $h(n)$ of the filter and sketch it. (6)

(iii) Find the frequency response $\hat{H}(f)$ and sketch its magnitude. What type of filter is it? (6)

(iv) The random input signal $x(n)$ consists of independent, zero mean, Gaussian distributed samples with power spectral density $R_{xx}(f) = \text{rect}(4f - 1)$, with $f \in [0, 1)$. Find the power spectral density $R_{yy}(f)$ of the output $y(n)$. (6)

(v) Find the power M_x and M_y at the system's input and output, respectively. (12)

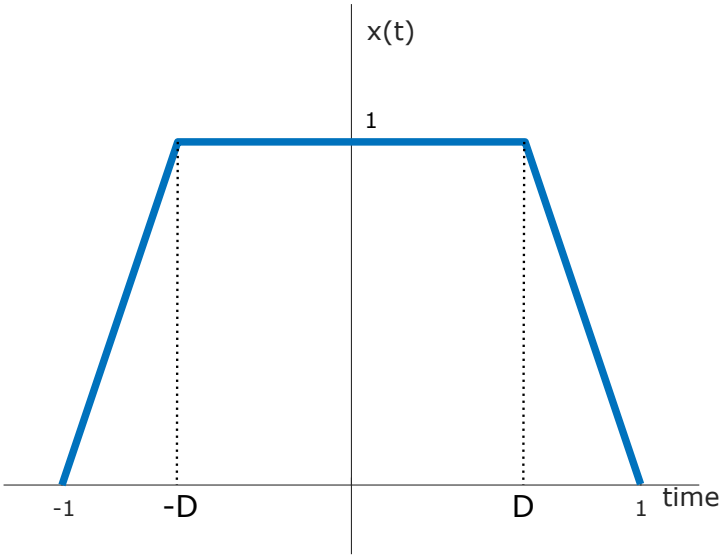

Figure 1: Signal $x(t)$