Digitale Signalverarbeitung
Tuesday 28 June, 2022

700.121 - SS 2022

Exam simulation
Answer the following questions. Total points: 100. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Series, Fourier Transform, Sampling Theorem** (40 points)
   - Let $x_p(t)$ be the periodic version of the signal illustrated in Fig.1, with $D \leq 1$ and period $T_0 = 2$:
     (i) Compute the Fourier Series of $x_p(t)$. (15)
     (ii) Having found that

$$x_p(t) = \frac{D}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(\frac{\pi Dk}{2}\right)}{k}\cos\left(\pi kt - \frac{\pi Dk}{2}\right),$$

     prove that

$$\sum_{k=1}^{\infty}\frac{\sin^2\left(\frac{\pi Dk}{2}\right)}{k^2} = \left(D - \frac{D^2}{2}\right)\frac{\pi^2}{4}. \quad (7)$$

     (iii) Compute the Fourier Transform $X(f)$ of $x(t)$ and sketch magnitude and phase. (5)
     (iv) Find $y(t) = x * x(t)$ the convolution of x with itself and evaluate the Fourier Transform $Y(f)$. (13)

2. **Discrete Fourier Transform** (10 points)
   - The DFT on an N-point signal $x(n)$ is given by

$$X(h) = [0, e^{j\theta}, \underbrace{0, 0, \ldots, 0}_{N-3 \text{ values}}, e^{-j\theta}], \;\; 0 \leq h \leq N-1$$

     (i) Find the N-point signal $x(n)$ and simplify your answer. (10)

3. **Z-Transform, Discrete Time System, Power Spectral Density** (50 points)
   - A very simple high-pass digital filtering can be performed using the following FIR filter:

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1}(-1)^k x(n-k)$$

     where $x(n)$ is the signal to filter. Assume $N = 8$.

(i) Sketch the impulse response $h(n)$ of the high-pass filter. (5)

(ii) What is the DC gain ($\hat{H}(0)$) of the filter? (5)

(iii) Find the transform function $H(z)$ and sketch the zero/pole diagram. (12)

(iv) Find the frequency response $\hat{H}(f)$ and sketch its magnitude. (10)

(v) The random input signal $x(n)$ is a moving average between two consecutive independent, zero mean, unit variance Gaussian distributed samples $w(n)$. Find the power spectral density $R_{yy}(f)$ of the output $y(n)$. (10)

(vi) Find the power M_y at the system's output. (8)

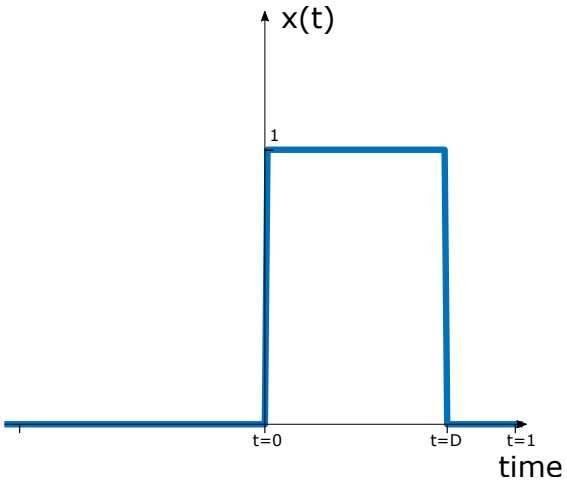

Figure 1: Signal $x(t)$.