Signal Processing for Communications

Thursday 6 February, 2020

700.602 - WS 2019

Written Exam

Answer the following questions. Total points: 110. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Transform, Sampling Theorem, Hilbert Transform** (40 points)
   - Let $x(t) = \sin^n(2\pi f_0 t)$ be a continuous-time signal with $n = 3$:
     (i) Find the period of $x(t)$. (3)
     (ii) Compute its Fourier Transform $X(f)$ and sketch the magnitude. (12)
     (iii) Having found that
     $$X(f) = -\frac{1}{8j}\Big(\delta(f-3f_0) - 3\delta(f-f_0) + 3\delta(f+f_0) - \delta(f+3f_0)\Big),$$
     compute $z(t) = x * y(t)$, with $y(t) = 4f_0\text{sinc}(4f_0 t)$, and sketch it. How is $z(t)$ related to $x(t)$? (10)
     (iv) What is the minimum sampling frequency F_c to guarantee a perfect reconstruction from the samples $x(kT)$? (3)
     (v) If $n = 5$, what would be the new minimum sampling frequency F_c? (5)
     (vi) Compute the Hilbert Transform $x_H(t) = \mathcal{H}\{x(t)\}$. (7)

2. **Random Process Theory, Power Spectral Density** (30 points)
   - Let $x(t)$ be a Gaussian process with zero mean and power spectral density $R_x(f) = R_0\text{triangle}(fT)$.
     (i) Evaluate the average power M_x. (5)
     The process is filtered through an interpolation filter with impulse response $h(t) = \frac{1}{T}\text{sinc}(\frac{t}{T})$, so that $y(t) = x * h(t)$:
     (ii) Evaluate the probability $P_1 = P[|y(t)| < V_0]$. (15)
     (iii) Evaluate the probability $P_2 = P[|y(t) + y(t-T)| < 2V_0]$. (10)

3. **Analog and Digital Modulation, Gram-Schmidt, Probability of Error** (40 points)
   - You are given the digital modulated signals in Fig.1.
     (i) Find an orthonormal base $\phi_i(t)$ for the set of signals $x_k(t)$ with $k = 1, 2, 3, 4$, where:

1) $x_1(t) = \sin(2\pi \frac{t}{T})$;

2) $x_2(t) = \sin(4\pi \frac{t}{T}) \cdot \cos(2\pi \frac{t}{T})$;

3) $x_3(t) = \sin(2\pi \frac{t}{T}) \cdot \cos(4\pi \frac{t}{T})$;

4) $x_4(t) = \sin^3(2\pi \frac{t}{T})$. (20)

Hint: Werner formulas

(ii) Sketch the transmitted constellation diagram. (10)

(iii) Evaluate the maximum probability of error of the system in the model $Y = X + N$ with N Gaussian noise with zero mean and variance σ^2. (5)

(iv) If a fifth signal $x_5(t) = \cos(2\pi \frac{t}{T})$ is transmitted, what would be a new orthonormal base $\hat{\phi}_i(t)$? (5)

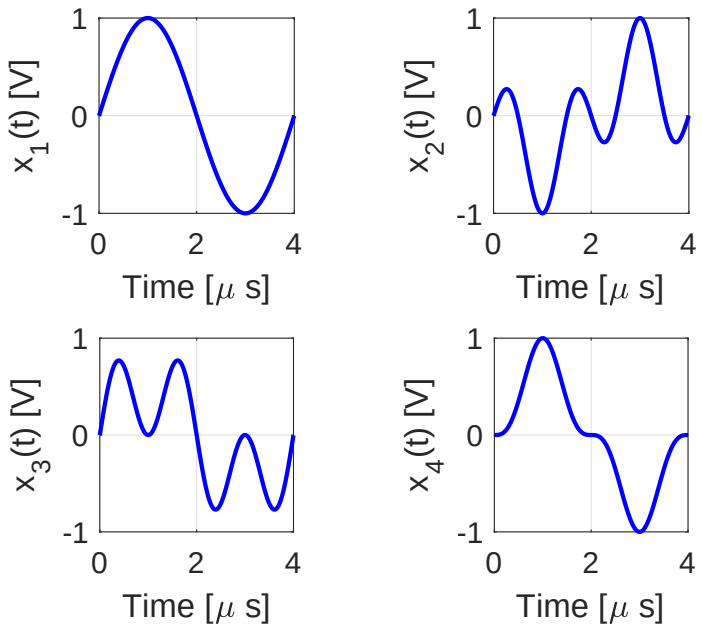

Figure 1: Modulated signals