Signal Processing for Communications

Monday 20 January, 2020

700.602 - WS 2019

Exam simulation

Answer the following questions. Total points: 100. Sufficient if ≥ 60. Time 1.5 hours.

1. **Fourier Transform, Sampling Theorem** (25 points)
   - Let $x(t) = \text{sinc}^n(2-3t)$ be a continuous-time signal with $n = 2$:
     (i) Sketch the amplitude of $x(t)$. (3)
     (ii) Compute its Fourier Transform $X(f)$ and sketch the magnitude. (10)
     (iii) Evaluate the energy of $X(f)$. (5)
     (iv) What is the minimum sampling frequency F_c to guarantee a perfect reconstruction from the samples $x(kT)$? (3)
     (v) If $n = 3$, what would be the new minimum sampling frequency F_c? (4)

2. **Random Process Theory, Power Spectral Density** (25 points)
   - Let X be a random variable with PDF given by

$$f_X(x) = \begin{cases} kx^2 & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

     (i) Find the constant k. (3)
     (ii) Compute the expected value $E[X]$ and the variance $V(X)$ of X. (10)
     (iii) Find the CDF $F_X(x)$ and sketch it. (5)
     (iv) Let $Y = \log(|X|)$ be a transformation. What is the expected value $E[Y]$ of Y? (4)
     (v) Find the PDF $f_Y(y)$ of Y and sketch it. (3)

3. **Complex Representations, Analytic Signal and Hilbert Transform** (25 points)
   - Let $x(t) = \text{sinc}(2t) + \sin(2\pi t)$ be a continuous-time signal:
     (i) Compute its Fourier Transform $X(f)$. (3)
     (ii) Compute its analytic signal $x^{(a)}(t)$. (12)
     (iii) Compute its Hilbert Transform $x_H(t)$ and sketch it. (3)
     (iv) Evaluate the energy of $x_H(t)$. (7)

4. **Analog and Digital Modulation, Gram-Schmidt, Probability of Error** (25 points)

   - You are given the digital modulated signals in Fig.1.

     (i) Find an orthonormal base $\phi_i(t)$ for the set of signals $x_k(t)$ with $k = 1, 2, 3, 4$. (8)

     (ii) Sketch the transmitted constellation diagram. (12)

     (iii) Evaluate the maximum probability of error of the system in the model $Y = X + N$ with N Gaussian noise with zero mean and variance σ^2. (5)

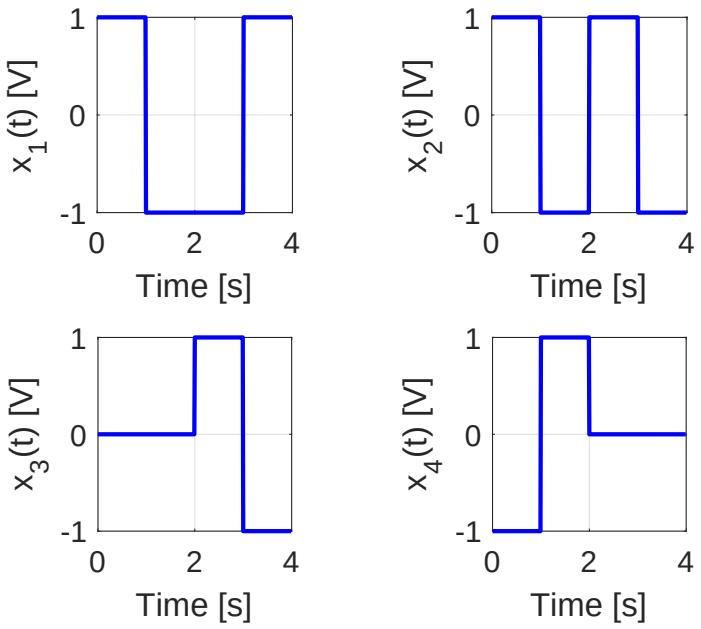

Figure 1: Modulated signals